CHAPTER X

ORTHOSCOPY

WE have already discovered in the pin-hole camera an optical instrument whose images are similar in their smallest details to the object reproduced and have now to investigate how far a photographic objective can produce a picture free from distortion. With the conclusions of the previous chapter in mind, we can give a general answer to this question.

CONDITION FOR FREEDOM FROM DISTORTION.— Let us project every solid object in the object space which is to be pictured, upon the focusing plane (Fig. 25), which is conjugate to the plane of the ground glass. Let us assume that the finite pencils are replaced by their principal rays. Let us, as before, designate the size of the object and the size of the image by y and y', the angles of inclination of the principal rays with the axis in the pupils on the object and image sides by ω and ω', and let the distances of the pupils from the focusing plane and ground-glass plane equal a and a'.

If the objective is to be free from distortion, the reduction factor $m = y : y'$ must remain constant for all values of the angle ω. The mathematical expression for this is:

$$\frac{y}{y'} = \frac{a \tan \omega}{a' \tan \omega'} = m.$$

This condition is independent of the amount of correction of the objective, and assumes only that every

oblique pencil can be represented by a principal ray.

BARREL-SHAPED AND CUSHION-SHAPED DISTORTION. — When the reduction factor m is not constant, we can distinguish two general cases. If the scale of reduction decreases with increasing size of the object, a square (Fig. 27, a) is imaged with its sides curved away from the center (Fig. 27, b). Such a distortion which is called *barrel-shaped*, is observed in the case of ordinary objectives with a front diaphragm.

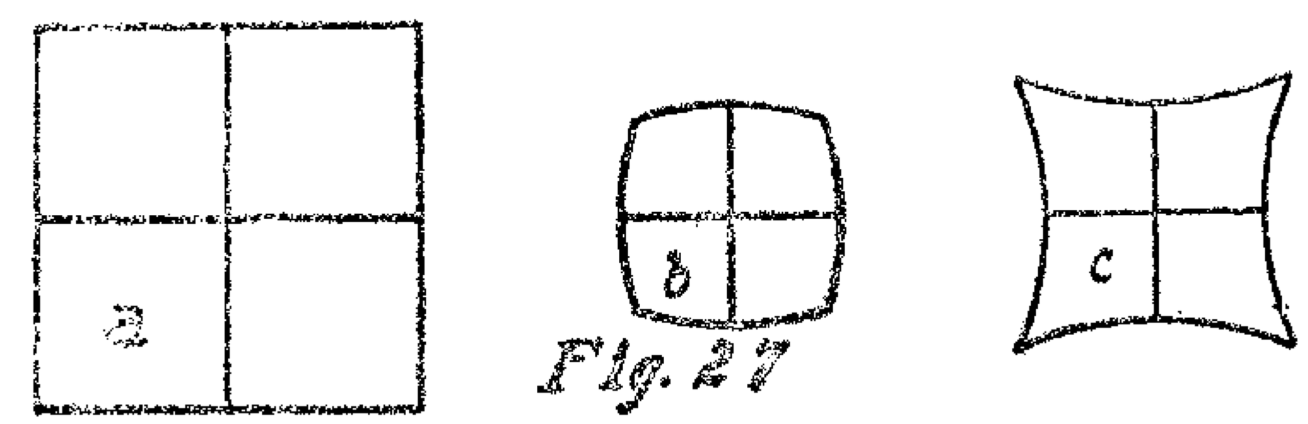

Fig. 27

If, on the other hand, m increases simultaneously with y, the image of the square has its sides concave. We observe this *cushion-shaped* distortion when we place the stop behind the lens. These phenomena become especially striking if we move the stop away from the lens, and thus modify the path of the rays.

BEHAVIOR OF THE SYMMETRICAL OBJECTIVE.— As we have already seen, the important class of symmetrical objectives is characterized by the fact that the principal rays before and after refraction form equal angles ω and ω' with the optical axis. The condition for orthoscopy then becomes $a : a' = m$. The relation of these distances of the focusing and ground-glass planes is, however, in general, not constant for any desired inclination of the principal rays, but only when $a = a'$. A symmetrical objective is, therefore, *absolutely* free from distortion only in the special case when the distances of the object and the image from the pupils are *equal*,— that is, when copying full size. In all other cases, the two pencils of principal rays intersecting in the pupils are not free from spherical aberration, so that in general we cannot say that the symmetrical objective is completely orthoscopic. Nevertheless, its deviations from orthoscopy are so slight that they do not become important, even for many tasks of reproduction. It is, however, very easily possible,— and, as a matter of fact, such corrections have been carried out for purposes of photogrammetry,— to give a symmetrical objective a higher degree of freedom from distortion by removing the aberrations in the pupils.

Zones of Distortion.— If the objective is not symmetrical, with a given reduction factor m, we must fulfil the general condition for a properly chosen angle of inclination of the principal ray.

If an objective is corrected for an inclination of the principal ray, ω, corresponding to the condition for orthoscopy, for smaller and larger angles there will occur in the usual way, zones, which, however, are practically unimportant. Zones also occur if we change the scale of reduction; thus, for example, if we move the object from a position near the objective to an infinite distance. These errors also are harmless in practice. We can assume in general that the modern types of photographic objectives in use at the present day are practically free from distortion, as long as they are used for the purposes intended by their designers and are not forced to cover too large a field. This is true even for the modern objectives with front diaphragms, which work at small apertures, and are used only for small sizes of plates; these are naturally never used for photogrammetry or reproduction.

Distortion of Convergence.— We assume in our consideration that the focusing plane, and, therefore,

the plane of the ground glass, are perpendicular to the optical axis. It is well known, however, that characteristic distortions of convergence appear in the image when the ground glass is placed at an angle to the focusing plane; these, however, have nothing to do with the above-mentioned deviations from orthoscopy. In such pictures of parallel arrangements we find the unpleasantly converging lines which are, for instance, apparent in architectural pictures, if the photographic apparatus was not placed exactly vertical. This divergence of lines can be removed by an extremely tedious method of copying.

Anamorphosis.— We will refer only briefly to the *anamorphous* lens systems in which a distortion is purposely produced, so that, for instance, a circle is transformed into an ellipse or a square into a rectangle. Such apparatus is valuable technically, especially in the preparation of designs. These transformations may, as long as the scale of reductions in the two directions perpendicular to each other is not very divergent, be produced also by means of a strongly diaphragmed lens or a pin-hole camera.

ground glass in all parts of the field which are to be used for the exposure, there will appear unsharp zones in the image which can only be removed by stopping down. The greater the aperture of the objective, the nearer the plane of the ground glass must the anastigmatic curves lie. Unsharp zones can also appear even if the median line between the two curves corresponds approximately with the plane of the ground glass, but at the same time the astigmatism goes beyond permissible limits. This also must be avoided in every anastigmat.

Extent of the Flat Field.— For how wide a field astigmatism and curvature of field must be thoroughly corrected, depends in general on the relative aperture of the objective. Systems of wide aperture, as a rule, cover smaller plates than narrow angles, but this comparison can only be made between anastigmats. It is most useful to calculate the anastigmatic correction not for the edge of the field, but for a point of lesser inclination to the principal ray, in order to minimize the zones in the middle of the field as far as possible, so that by stopping down a considerably larger field can be covered.

That in the case of difficult exposures of a plane object perpendicular to the axis, one must not focus exclusively on the middle of the field, but seek to obtain a compromise between the different parts of the field, is well known.

CHAPTER IX

Limitation of the Rays

In the previous chapters we have often introduced the idea of stopping down. We will now consider, although briefly, the influence of the introduction of a diaphragm on the course of the rays. We cannot, however, attempt to include a comprehensive treatment of Abbe's theory of the limitation of rays.

The Diaphragm.— Every optical system must have some sort of physical bounds. In photographic objectives this occurs through the introduction of a circular aperture whose center lies in the optical axis. This is the *diaphragm*. By the introduction of this into the course of the light rays, a definite pencil is separated, which, according to its area of cross-section, produces a more or less bright image. In order to be able to vary the amount of illumination for various purposes, we use diaphragms of different diameters, which can be either gradually changed (iris diaphragms), or varied by definite steps (sets of stops, or in case the apertures are arranged in *one* piece of metal, the rotating stop). For certain special tasks in photo-engraving, there may be arranged in *one* diaphragm several apertures, whose position and shape must correspond to the structure of the screen. These are *Grebe's coincidence diaphragms*.

The *stop* (B, Fig. 25), as it is also called, is placed between the members of the objective in compound objectives. The principal rays of the pencils inclined to the optical axis go through its center, and therefore every principal ray, *before* its refraction by the front member of the objective, L_1, must tend toward a point which forms the center of the virtual image of the stop B produced by the lens L_1. In exactly the same way every principal ray, *after* its refraction by the rear lens L_2, appears to come from the center of the virtual image of the stop B produced by the lens L_2.

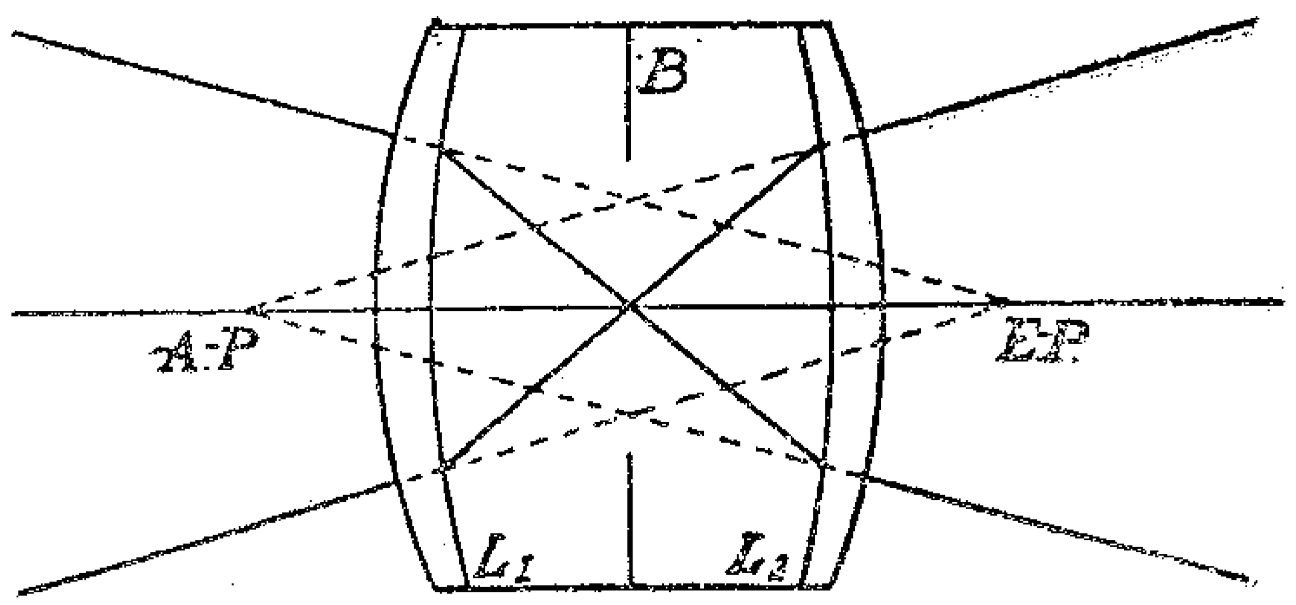

Fig. 25

PUPILS.— These two centers of the virtual images are called by Abbe the *pupils;* the incident principal ray is directed toward the center of the *entrance-pupil* (E.-P., Fig. 26), while the principal ray, after its emergence, appears to come from the center of the *exit-pupil* (A.-P.). The entrance- and exit-pupils are, therefore, the bases of the incident and emergent light pencils whose apices lie at the object point and the image point. Instead of the word diaphragm, one often uses *iris* in comparison with the construction of the human eye.

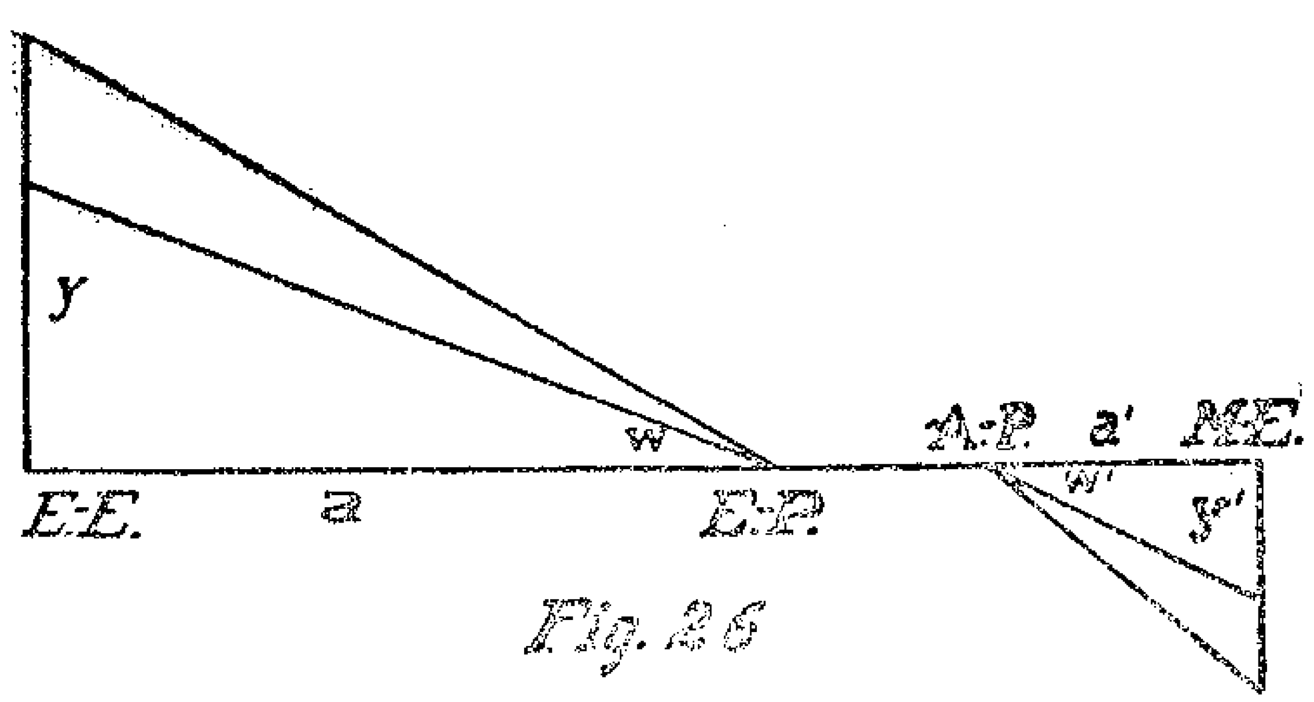

Fig. 26

OBJECTIVES WITH FRONT STOPS.— In the case of objectives with stops in front of the lens (often called *landscape lenses*), as, for instance, the rear half of a symmetrical objective, the stop coincides with the entrance-pupil. The exit-pupil is here virtual. We shall later see how the external cylindrical surfaces of photographic objectives, which also form stops, act on the course of the ray.

SOLID OBJECTS.— In point-image formation an image plane corresponds to every object plane, and it is not possible to form a sharp image of even an infinitely small portion of space in *one* plane. We have, however, in scientific, technical, and artistic photography, almost always to deal with solid objects, whose single points, on account of their different distances from the camera, must produce images in various planes of the image space. If we should insist upon a strict point-image formation, and require that all aberrations must be completely removed, every photographic process would be out of the question, for we can actually use in the image space only a *single* plane upon which the point-image formation must occur, while, as a matter of fact, the images lie in different planes.

THE FOCUSING PLANE CONJUGATE TO THE PLANE OF THE GROUND GLASS.— We can simplify the conception considerably if we translate all the space phenomena of the object space to a *single* plane, as proposed by M. von Rohr. This *focusing-plane* (E.-E.) has as its image the *plane of the ground glass*. If these two planes are determined by sharp focusing of the objective on a given point, the image formation proceeds as if the whole object space were replaced by this plane, upon which all objects in the image space are projected by means of the principal rays from all the object points to the center of the entrance-pupil. The principal rays, going from the exit-pupil in the image space, then intersect the ground glass plane (M.-E.) in the image points which are conjugate to the various points of the projection on the focusing plane.

PROJECTIVE PROPERTIES OF THE PUPILS.— From this it follows that if the figure produced by a projection of the solid objects on the focusing plane is viewed from the center of the entrance-pupil, or if the image is viewed from the center of the exit-pupil, they appear in *central projection*. The *centers of the pupils* are, therefore, the *centers of perspective*. If we assume that the double objective within which the diaphragm is placed becomes infinitely thin, the stop coincides with the two pupils, and becomes the center of projection, as the pin-hole does in its camera.

PRINCIPAL RAYS AND PRINCIPAL POINTS.— If the objective is symmetrical and the diaphragm is placed in the center of the air-space between the two halves, then the centers of the pupils are identical with the principal points for very small inclinations of the principal rays. Except in this single case, however, the points of intersection of the principal rays with the axis before and after refraction occupy somewhat different positions from the principal points.

VIEWING DISTANCE.— It is apparent that the purpose of every photographic exposure is the production of a picture which is in all its parts exactly similar to the projection of the external world upon the focusing plane. The viewing of this photographic picture, however, must, if it is to make a truthful impression upon the eye, include the same angle which the corresponding principal rays form at the center of the entrance-pupil. A simple mathematical calculation shows that this condition if fulfilled if we view the picture from a distance equal to the *focal length of the taking lens*. It is true that this rule is exactly valid only when the focusing plane was infinitely distant from the objective; however, the deviation of the viewing distance from this limiting case in hand-camera exposures, which give a considerably reduced image, is so slight that we can neglect it. If a picture, therefore, is to be viewed at the normal viewing distance of an average eye, the focal length of the objective must be about 25 cm (10 in.), if the impression is to be true to nature. If the focal length is less, the eye must be brought nearer to the picture.

VIEWING LENSES FOR PHOTOGRAPHIC PRINTS.— Since, however, the accommodation of the eye renders it impossible to use much shorter distances for viewing such pictures, we must use, between the eye and the print, a *collecting lens* which must fulfil two conditions. In the first place the picture viewed through this lens must appear to be at the same distance which the object actually had when taken, and secondly it is absolutely necessary that the picture should appear in correct perspective.

FOCAL LENGTH OF THE VIEWING LENS.— If we state these conditions in the shape of a very simple mathematical formula, we immediately obtain the interesting law that every photographic picture should be viewed through a collecting lens whose focal length is equal to that of the taking objective, and from a distance equal to this focal length. One may easily convince himself of the truth of this law by viewing one of the well-